Even-Mode Characteristics of the Bilateral Slotline

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Abstract—**Transmission line data are presented for the even-mode characteristics of a bilateral slotline. The bilateral slotline (double-sided slotline) is analyzed by the spectral Galerkin method and results are presented for the slot wavelength and characteristic impedance for substrates having $\epsilon_r = 2.22$, 3.5, 6.0, 9.8, and 12.8. The results are presented for $0.01 \leq \sqrt{\epsilon_r - 1}\, d/\lambda_0 \leq 0.25$, where $2d$ is the substrate thickness and ϵ_r its dielectric constant.**

I. Introduction

SINGLE-SIDED slotlines have been in use for quite some time as transmission lines and circuits [1]. More recently, however, slotlines have increasingly been utilized in the development of wide-band microwave and millimeter-wave antennas (see [2] for a recent summary). A microstrip-to-slotline or a coaxial-to-slotline transition is often needed to couple energy from the feeding line to the radiating slotline antenna. The resulting structure has the disadvantage of having a radiating feed. In order to reduce radiation loss from the feed, the slotline antenna can be designed using a double-sided board and can be fed by means of a nonradiating feed such as a stripline. Such a structure was considered in [3] and [4]. A double-sided slotline has two identical slots that are on opposite sides of a dielectric substrate and are arranged one on top of the other without any lateral displacement. We call such a transmission line a bilateral slotline (The nomenclature is borrowed from a similar structure found in a finline version.)

In this paper we analyze the bilateral slotline and present its even-mode characteristics. Only the dominant mode is considered. Results are presented for the slot wavelength and the characteristic impedance for $\epsilon_r = 2.22$, 3.5, 6.0, 9.8, and 12.8. The computed results for $\epsilon_r = 2.22$ are compared with results available in the literature [5] for a shielded bilateral slotline. The author of [5] has performed an asymptotic analysis based on the Wiener–Hopf technique. Data were presented only for $\epsilon_r = 2.22$ and for wide slots. Comparison is made for this case to validate the computed data. The problem is formulated and solved using the spectral Galerkin's method. Over the past two decades, the spectral-domain method has emerged as a powerful tool for analyzing planar transmission and radiation problems. Theory pertaining to this method is available elsewhere in the literature [6]. In Section II, we present the theory and outline only the main steps leading to the formulation of the problem. In Section III we present and discuss the numerical results. Calculation of conductor loss is not presented here. However, it may be determined by assuming finite conductor thickness and employing the technique of [10] to avoid the nonintegrable singularity at the edge, as is done in [11] for coplanar waveguides.

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II. Formulation of the Problem

Fig. 1 shows the geometry of the bilateral slotline. It consists of symmetric slots of width w etched on opposite sides of a dielectric substrate of thickness $2d$ and having a relative permittivity ϵ_r. The structure has a plane of symmetry shown as a dashed line in Fig. 1. Because of the symmetry, two modes can propagate on the structure—an odd mode and an even mode. The odd mode is the one in which the tangential electric field in the upper slot is equal in magnitude but opposite in sign to the tangential electric field in the lower slot. The field configuration is the same as that in an electric conductor-backed single-sided slotline with the conductor placed in the plane of symmetry. Such a case has already been treated in the literature [7] and will not be considered here. The even mode is the one in which the slot electric field is identical in both the upper and the lower slot. Such a mode is supported by a perfect magnetic conductor placed in the plane of symmetry. It is the even mode that will be the topic of discussion of this paper. In the ensuing analysis, we will assume that the structure under consideration consists of a single-sided slot of width w etched on a magnetic conductor-backed substrate of thickness d.

An important distinction exists between the electric conductor-backed slotline and the magnetic conductor-backed slotline. In an electric conductor-backed slotline, power could leak away from the slot in the lateral direction in the form of a parallel-plate waveguide mode [7]. Far away from the slot, the structure resembles a conventional dielectric-filled parallel-plate waveguide formed by placing electric conductors on both sides of the substrate. This waveguide supports a TEM mode for which there is no cutoff frequency. The leaky TEM mode must be con-

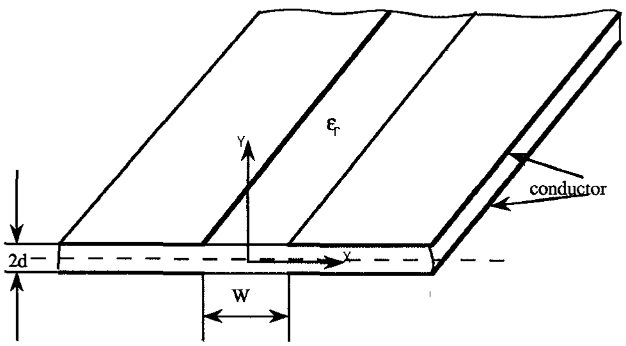


Fig. 1. Geometry of the bilateral slotline.

sidered an integral part of the slotline mode during the analysis [7]. However, in the magnetic conductor-backed slotline, the dielectric-filled parallel-plate waveguide has an electric conductor on one side (plane containing the slot) but a magnetic conductor on the other side of the substrate. The dominant mode in this structure has a cutoff frequency and will not be excited as long as $\sqrt{\epsilon_r}d/\lambda_0 < 0.25$, where λ_0 is the free-space wavelength. That this is the case can be easily verified by solving the waveguide problem pertaining to the above geometry. We will show shortly that a slightly thicker substrate can be considered while ensuring that only the slotline mode will propagate in the structure.

An $e^{j(\omega t - \beta z)}$ time and z dependence is assumed for all field quantities and suppressed throughout. In the above expression ω in the radian frequency and β is the unknown propagation constant yet to be determined. Following the standard spectral-domain approach, the slot electric field (E_x, E_z) can be related to the electric surface currents (J_x, J_z) as

$$\begin{pmatrix} G_{xx} & G_{xz} \\ G_{zx} & G_{zz} \end{pmatrix} \begin{pmatrix} \tilde{E}_x(\alpha) \\ \tilde{E}_z(\alpha) \end{pmatrix} = \begin{pmatrix} \tilde{J}_x(\alpha) \\ \tilde{J}_z(\alpha) \end{pmatrix}. \tag{1}$$

A tilde denotes quantities Fourier-transformed with respect to x, and α is the transformed variable. In the above expression, G_{xx}, G_{xz}, G_{zx}, and G_{zz} are elements of the dyadic Green's function, given by

$$G_{xx} = \cos^2\theta G_{11} + \sin^2\theta G_{22}$$
$$G_{xz} = G_{zx} = \sin\theta\cos\theta(G_{22} - G_{11})$$
$$G_{zz} = \sin^2\theta G_{11} + \cos^2\theta G_{22}$$

where $\sin\theta = \alpha/\sqrt{\alpha^2+\beta^2}$, $\cos\theta = \beta/\sqrt{\alpha^2+\beta^2}$, and

$$G_{11} = \frac{-1}{j\omega\mu_0}(\gamma_0 + \gamma_2 \tanh\gamma_2 d)$$

$$G_{22} = -j\omega\epsilon_0\left(\frac{1}{\gamma_0} + \frac{\epsilon_r \tanh\gamma_2 d}{\gamma_2}\right)$$

where $\gamma_0 = \sqrt{\alpha^2+\beta^2-k_0^2}$, $\gamma_2 = \sqrt{\alpha^2+\beta^2-\epsilon_r k_0^2}$, and $k_0 = \omega\sqrt{\epsilon_0\mu_0}$ is the free-space wavenumber. The zeros of G_{11} and G_{22} in the α plane give the location of TE_y and TM_y modes on a magnetic conductor-backed dielectric slab. The poles of G_{11} and G_{22} represent the modes of a parallel-plate waveguide formed by a magnetic conductor on one side and an electric conductor on the other side of a dielectric slab of thickness d. The poles will occur at $\gamma_2 d = j(p-1/2)\pi$, $p = 1, 2, \cdots$. Since $k_0 < \beta < \sqrt{\epsilon_r}k_0$, it is seen that the poles will be off the real axis in the α plane as long as $\sqrt{\epsilon_r - 1}\,d/\lambda_0 < 0.25$. This condition will be assumed throughout the paper.

Equation (1) is solved by the method of moments. The slot electric field is expanded in terms of known basis functions $[f_n(x), g_n(x)|$ as

$$E_x(x) \approx \sum_{n=1}^{N} a_n f_n(x) \qquad E_z(x) \approx \sum_{m=1}^{M} b_m g_m(x).$$

Galerkin's method of testing is used to discretize (1) to a set of homogeneous linear equations. The elements of the matrix are all functions of the unknown β. The propagation constant is obtained by solving for values of β that render the determinant zero.

The characteristic impedance, Z_0, of the slotline can be defined as [8]

$$Z_0 = \frac{|V_0|^2}{2P_f} \tag{2}$$

where V_0 is the voltage across the slot and P_f is the real part of the complex power. The voltage V_0 is obtained in terms of the transverse electric field as

$$V_0 = \int_{-w/2}^{w/2} E_x\,dx = \tilde{E}_x(0)$$

and the complex power P_f is obtained as [8]

$$P_f = \frac{1}{2\pi}\iint_{\alpha, y>0}\left(\tilde{E}_x\tilde{H}_y^* - \tilde{E}_y\tilde{H}_x^*\right)d\alpha\,dy. \tag{3}$$

The factor of 2 in the denominator of (2) accounts for the fact that an equal amount of power is carried by the lower slot.

III. Numerical Results and Discussion

The basis functions employed here are the same as those in [9], viz.,

$$f_n(x) = \frac{2}{\pi w}\frac{T_{2(n-1)}\left(\frac{2x}{w}\right)}{\sqrt{1-(2x/w)^2}}$$
$$\Leftrightarrow (-1)^{n-1}J_{2(n-1)}\left(\frac{\alpha w}{2}\right), \qquad n = 1, \cdots, N \tag{4a}$$

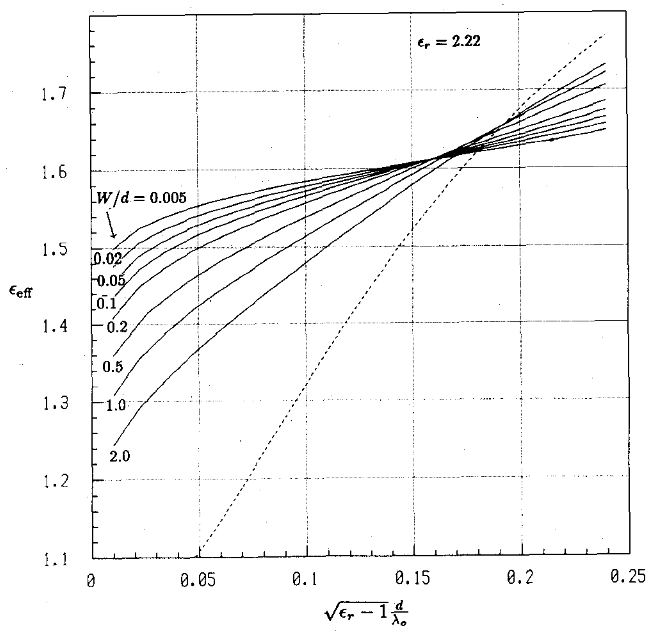
$$g_m(x) = \frac{2}{\pi w}\sqrt{1-(2x/w)^2}\,U_{2m-1}\left(\frac{2x}{w}\right)$$
$$\Leftrightarrow j\frac{(-1)^{m-1}2mJ_{2m}\left(\frac{\alpha w}{2}\right)}{\left(\frac{\alpha w}{2}\right)}, \qquad m = 1, \cdots, M \tag{4b}$$

where $T_n(x)$ and $U_n(x)$ are Chebyshev polynomials of order n of the first and the second kind respectively, and $J_n(x)$ is the Bessel function of the first kind of order n.

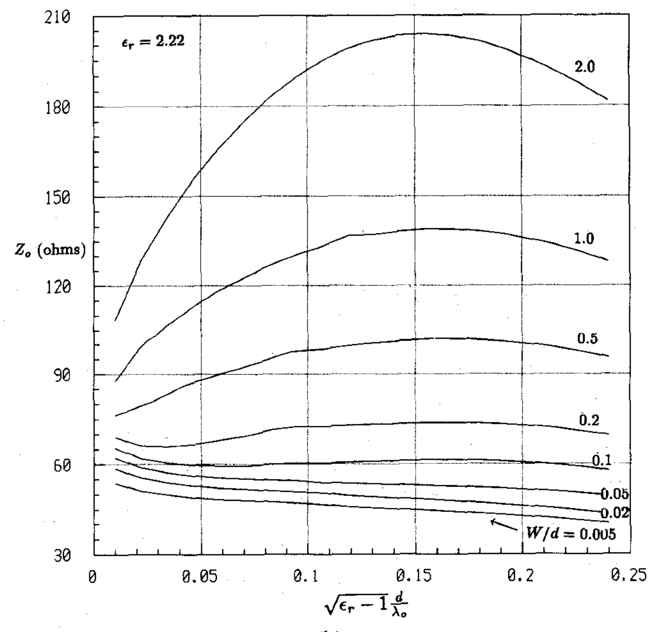
TABLE I
COMPARISON OF CALCULATED SLOTLINE DATA

		ϵ_{eff}		Z_o (ohms)	
d/λ_o	w/d	Present	Ref. [5]	Present	Ref. [5]
0.026	1	1.37	1.36	103.2	87.5
	4	1.24	1.20	186.9	184.8
0.107	1	1.54	1.56	139.6	119.0
	4	1.49	1.48	308.3	266.0
0.171	1	1.65	1.65	137.3	125.0
	4	1.65	1.65	315.4	309.0

$\epsilon_r = 2.22$, $d_o/d = 10$, d_o is the height of the shield above the plane of the slot.

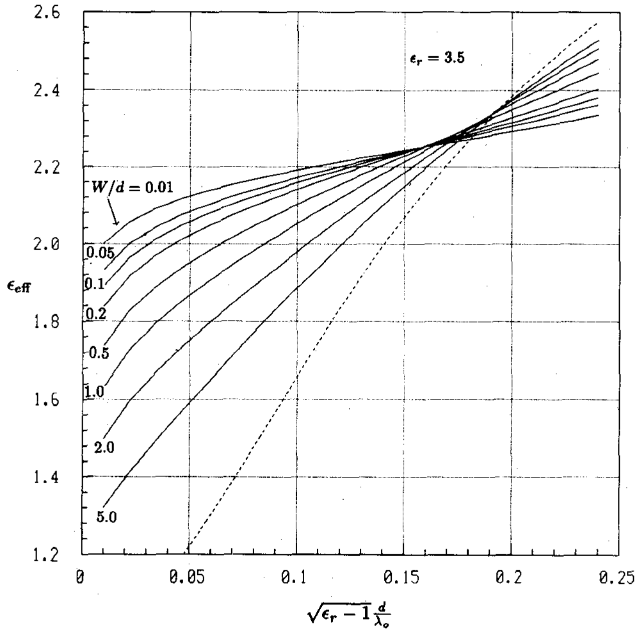


(a)

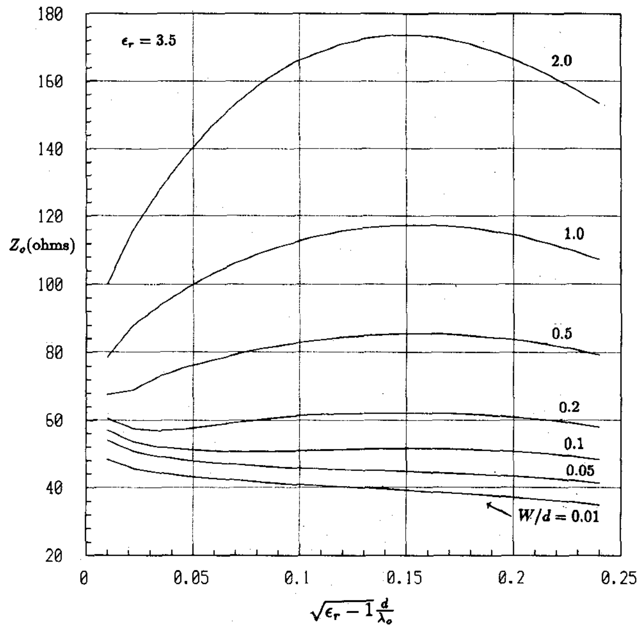


(b)

Fig. 2. (a) Dispersion characteristics of bilateral slotline. (b) Characteristic impedance of bilateral slotline. $\epsilon_r = 2.22$.



(a)



(b)

Fig. 3. (a) Dispersion characteristics of bilateral slotline. (b) Characteristic impedance of bilateral slotline. $\epsilon_r = 3.5$.

The above choice of basis functions facilitates extraction of the asymptotic contribution of the various integrals used to fill the matrix corresponding to (1). Integration with respect to y in (3) can be done in a closed form, thus converting it to a one-dimensional integral. A computer program was developed to compute the slot wavelength and the characteristic impedance of the bilateral slotline for a given set of substrate parameters and at a specified frequency. Numerical experiments were performed to study the convergence of the results with respect to the number of basis functions used. It was found that a convergent solution is found in most cases with just one basis function for E_x and none for E_z. However,

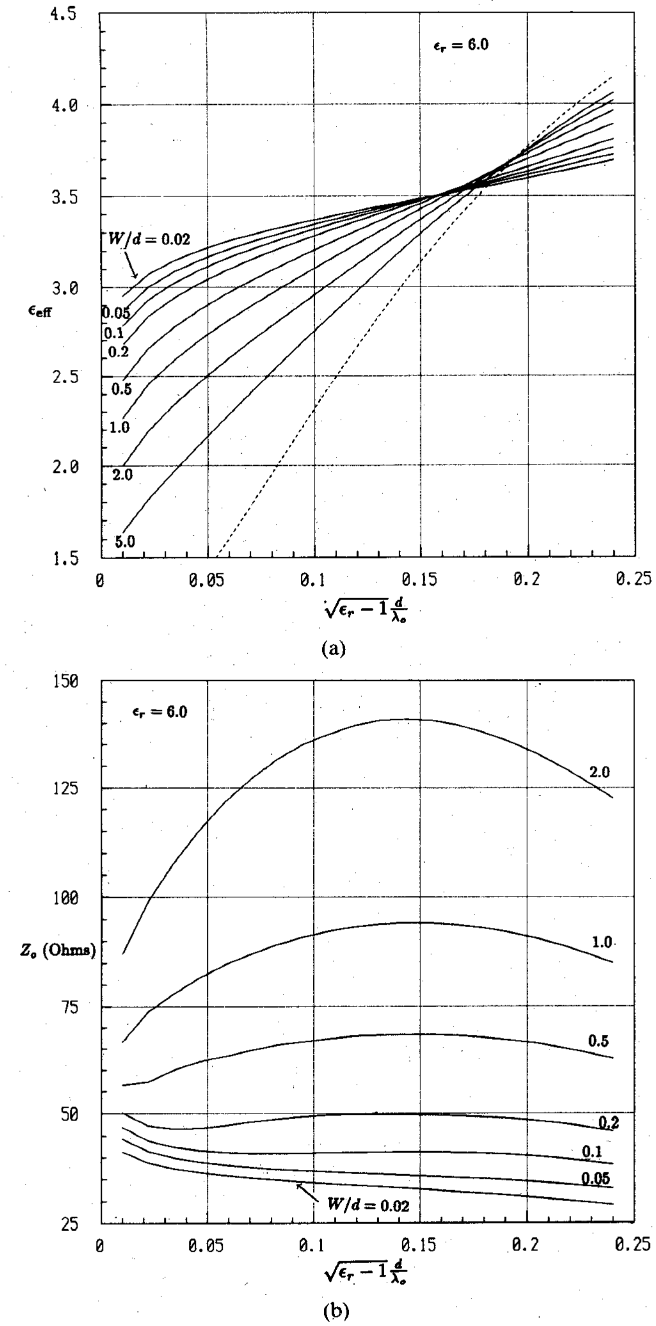


Fig. 4. (a) Dispersion characteristics of bilateral slotline. (b) Characteristic impedance of bilateral slotline. $\epsilon_r = 6.0$.

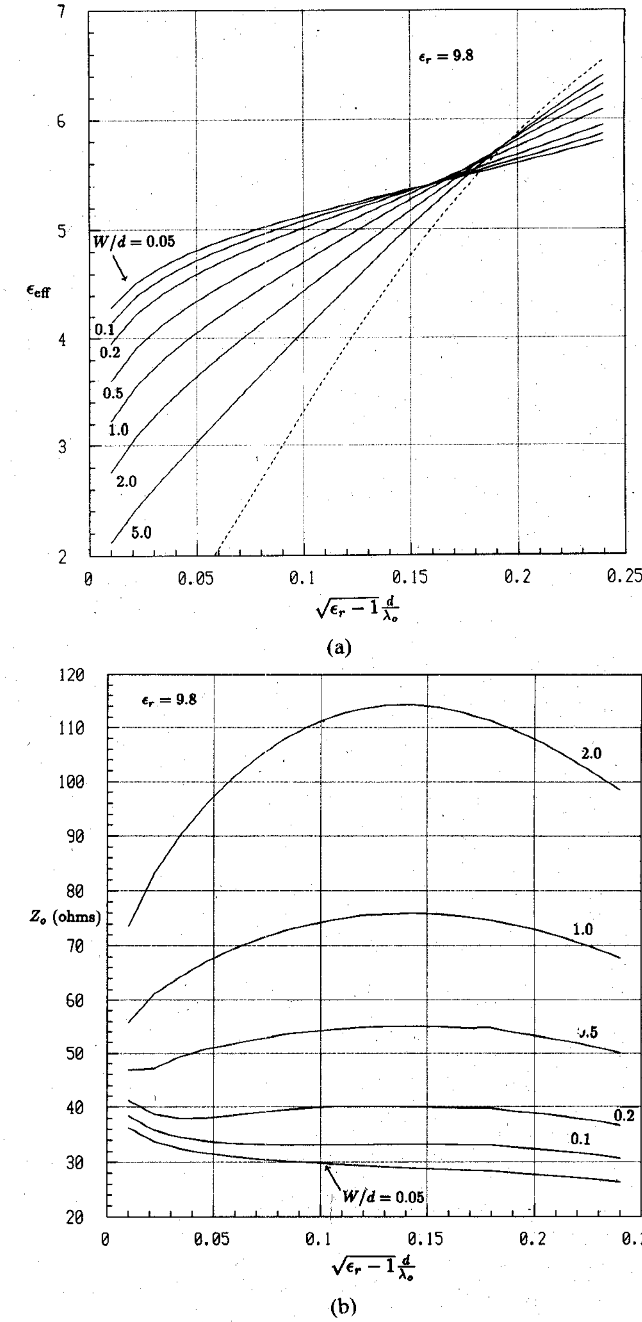


Fig. 5. (a) Dispersion characteristics of bilateral slotline. (b) Characteristic impedance of bilateral slotline. $\epsilon_r = 9.8$.

wider slots $(w/d) > 1$, in general, require additional basis functions for obtaining convergence. For all the cases presented in this paper, no more than four modes were used for E_x and no more than three modes were used for E_z.

In order to check the results obtained using the present code, data for $\epsilon_r = 2.22$ were compared with the corresponding data available in [5]. The author of [5] performed an asymptotic analysis based on the Wiener–Hopf theory on a *shielded* bilateral slotline and presented data for wide slots $w/d > 1$ and for $\epsilon_r = 2.22$. Table I shows a comparison between the two for the effective dielectric constant $\epsilon_{eff} = (\beta/k_0)^2$ and the characteristic impedance Z_0. It is seen that there is good agreement between the two for ϵ_{eff}, although entirely different techniques were used to arrive at the results. Disagreement in the data for Z_0 is due to the fact that in [5] a voltage–current definition was used, whereas in the present work a power–voltage definition is used.

The slot wavelength and the characteristic impedance were computed for some of the most commonly used substrates: $\epsilon_r = 2.22$, 3.5, 6.0, 9.8, and 12.8. The thickness of the substrate in each case ranged over $0.01 \leqslant \sqrt{\epsilon_r - 1}\, d/\lambda_0 < 0.25$. The results are plotted in Figs. 2–6. In each case, results are shown for $\epsilon_{eff} = (\beta/k_0)^2$ and Z_0. The dashed line in Figs. 2(a) through 6(a) repre-

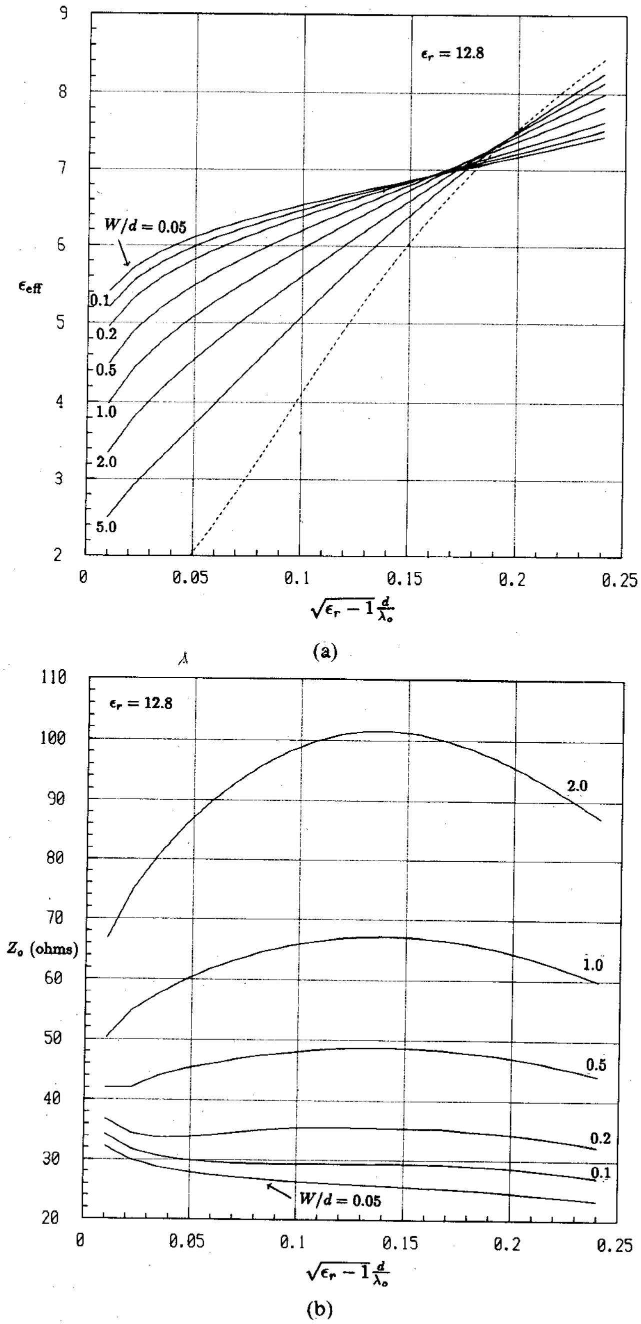


Fig. 6. (a) Dispersion characteristics of bilateral slotline. (b) Characteristic impedance of bilateral slotline. $\epsilon_r = 12.8$.

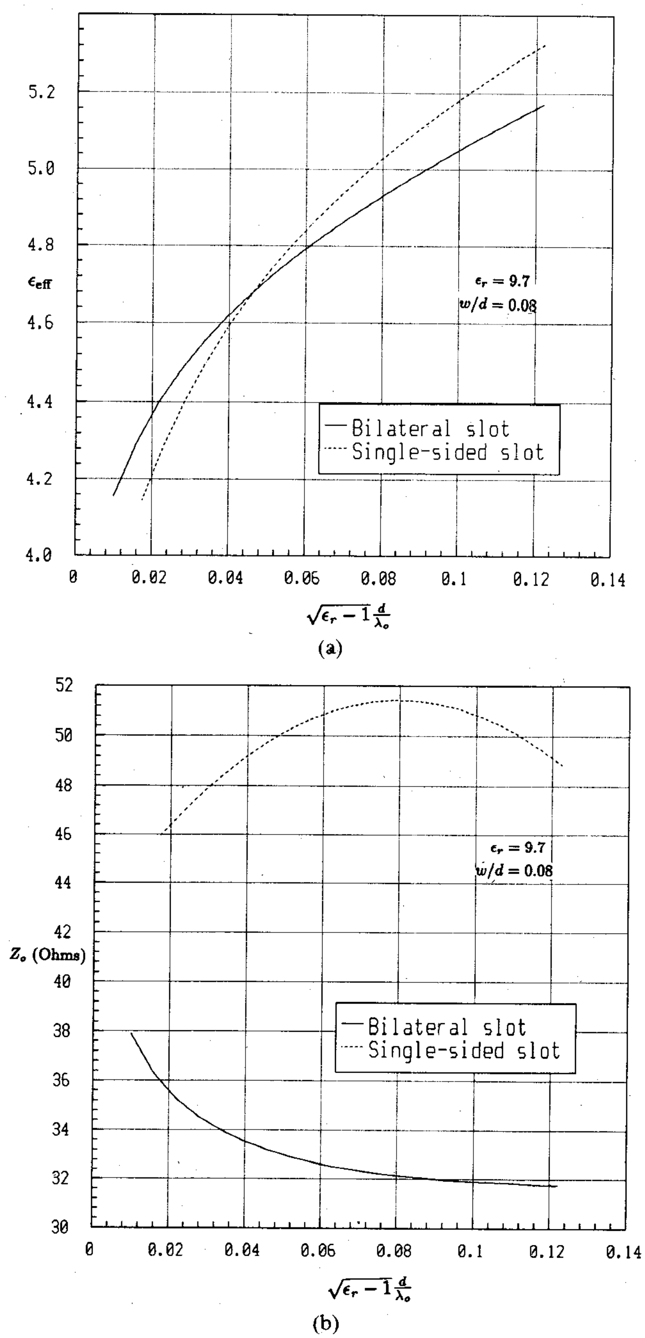


Fig. 7. Comparison of data between bilateral slotline and single-sided slotline. (a) ϵ_{eff}. (b) Z_0. $\epsilon_r = 9.7$, $w/d = 0.08$.

sents the propagation constant of the first TE_y mode on a magnetic conductor-backed dielectric slab. It is obtained by finding the first zero of G_{11}. In [5] the author claimed that the dispersion curves of the bilateral slotline intersect at a common point on the dispersion curve of this first TE_y mode *regardless of the width*. Such an effect was not observed in the present work. In [5] data were presented only for $\epsilon_r = 2.22$ and for $w/d \geqslant 1$. It is seen from Fig. 2(a) that for $\epsilon_r = 2.22$ and $w/d \geqslant 1$, the dispersion curves do intersect at a common point on the dispersion curve of the TE_y mode consistent with [5]. The common point of intersection reported in [5] was at $\sqrt{\epsilon_r}\,2\pi d/\lambda_0 = 1.6$, which is consistent with the value $\sqrt{\epsilon_r - 1}\,d/\lambda_0 = 0.19$ seen from Fig. 2(a). However, for $w/d < 1$, the dispersion curves do not intersect at the same point. Such behavior was found for all the data presented in this paper. Notice that for narrow slots, the impedance first decreases before increasing with increasing values of substrate thickness. This is seen more prominently for higher ϵ_r.

On a final note, we compare the characteristics of a bilateral slotline with those of a single-sided slotline for the same set of substrate parameters. Fig. 7 shows a comparison between the two for $\epsilon_r = 9.7$ and $w/d = 0.08$. In either case $2d$ is the total substrate thickness. Data for the single-sided slotline have been obtained by using the closed-form expressions provided in [1]. We see that the bilateral slotline has relatively less dispersion than the single-sided slotline for the same substrate parameters and width w. Fig. 7(b) shows a comparison of the impedance. Since the slot is rather narrow, the impedance of the bilateral slotline decreases with increasing substrate thickness, as seen from the figure. Further it is seen from Fig. 7(b) that the characteristic impedance of bilateral slotline is less than that of a single-sided slotline, the former being approximately half the latter. Since the characteristic impedance increases with slot width for a fixed substrate, this means that the slot width to achieve a given impedance is larger for the bilateral slotline than for the single-sided slotline. This partially overcomes the fabricational difficulty associated with etching narrow slots on low-permittivity substrates that is normally encountered in a single-sided slotline.

IV. Conclusion

A spectral-domain technique has been used to obtain the transmission line characteristics of the bilateral slotline. A computer program was developed to compute the slot wavelength and the characteristic impedance for a given set of substrate parameters and frequency. Comparison was made for the computed results with data available in the literature for $\epsilon_r = 2.22$. Results were presented for ϵ_{eff} and Z_0 for $\epsilon_r = 2.22$, 3.5, 6.0, 9.8, and 12.8. The results are valid for substrate parameters within the range $0.01 \leqslant \sqrt{\epsilon_r - 1}\, d/\lambda_0 < 0.25$.

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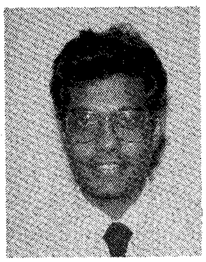

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